

Maintaining the local critical temperature of thermally out-of-equilibrium superconducting wires below the critical value

Y. Dubi and M. Di Ventra

Department of Physics, University of California–San Diego, La Jolla, California 92093-0319, USA

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A generalized theory of open quantum systems combined with mean-field theory is used to study a superconducting wire in contact with thermal baths at different temperatures. It is shown that, depending on the temperature of the colder bath, the temperature of the hotter bath can greatly exceed the equilibrium critical temperature, and still the wire remains in the superconducting state, meaning that the effective local temperature in the wire is maintained below the critical temperature. The effects of contact areas and disorder are studied. Finally, an experimental setup is suggested to test our predictions.

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Ever since the discovery of superconductivity, fabricating a superconducting (SC) wire that conducts electricity without dissipation at room temperature has been a major goal of modern condensed matter physics. However, since the discovery of high- T_c superconductors¹ (which have T_c as high as ~ 150 K, still far from room temperature and perhaps close to the upper limit in these materials²) there has been little or no progress in increasing T_c . Recently, several suggestions have been put forward to increase T_c by fabricating nanoclusters^{3,4} or layering of the SC material.^{5–7}

In this paper we consider a different route, based on local cooling, which may allow for clean SC wires to remain superconducting even if the external temperature exceeds their T_c . To this aim we study a SC wire in contact with two different heat baths, held at different temperatures. We consider at first a wire in contact with two heat baths at its edges, held at different temperatures T_L (left bath) and T_R (right bath), with $T_R > T_c^{eq} > T_L$, where T_c^{eq} is the equilibrium critical temperature [upper panel of Fig. 1(a)]. We find that depending on the value of T_L , the critical value $T_{R,c}$ (defined as the maximum value of the temperature T_R of the hot bath at which the wire is still SC) can be much larger than T_c^{eq} , which implies that the internal temperature in the wire is maintained below T_c^{eq} although the average of T_L and T_R exceeds it. We then study the effect of different couplings to the baths and of disorder on the above result. We find the dependence of $T_{R,c}$ on the coupling to the different baths, and demonstrate that, in agreement with Anderson's theorem,⁸ weak disorder does not change $T_{R,c}$ by much. Strong disorder leads to the breakdown of the SC state near the right (hot) bath, and to a spatial dependence of the SC order parameter.

Normal metallic wires in contact with two heat baths at the edges were recently studied in detail, in the context of heat flow in such systems.^{9–11} One of the main conclusions of Ref. 10 was that in a nonequilibrium system which is not diffusive, its energy distribution function (DF) is the average of that of the two baths (a similar observation was verified experimentally for short wires, when interactions are relatively unimportant,¹² a situation which is also likely to hold for the quasiparticles in SC wires). This observation will allow us to provide an analytic expression for the critical temperature $T_{R,c}$ which shows excellent fit with our numerical calculations, and provides a direct prediction which may be tested experimentally.

The method we use is a generalization of the Bogoliubov–de Gennes (BdG) mean-field theory¹³ to non-equilibrium. The starting point is the tight-binding BdG Hamiltonian on a square lattice (with lattice constant $a=1$), $\mathcal{H}_{BdG} = \sum_{i,\sigma} (\epsilon_i - \mu) c_{i\sigma}^\dagger c_{i\sigma} - t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + \sum_i (\Delta_i c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \text{h.c.})$, where $c_{i,\sigma}^\dagger$ creates an electron in the i th lattice site with spin σ , t is the hopping integral ($t=1$ serves as the energy scale hereafter), ϵ_i are random on-site energies drawn from a uniform distribution $U[-W/2, W/2]$ (hence W is the strength of disorder, with $W=0$ representing a clean system), μ is the chemical potential and Δ_i is the SC order parameter in the i th lattice site. The order parameter is to be determined self-consistently on every site via $\Delta_i = -U \langle c_{i\downarrow} c_{i\uparrow} \rangle$, where $U > 0$ is the effective electron-electron attractive interaction and $\langle \cdot \rangle$ stands for a statistical average. In this paper we treat only s -wave superconductors, but the formalism can easily be extended to account for other kinds of symmetry.

Since the BdG Hamiltonian is quadratic, it can be exactly diagonalized to describe the quasiparticle excitations $\gamma_{n\sigma}$. To diagonalize it, one performs a Bogoliubov transformation¹³ for the electron operators, $c_{i\sigma}^\dagger = \sum_n [u_n(i) \gamma_{n\sigma}^\dagger + \sigma v_n^*(i) \gamma_{n\bar{\sigma}}]$,

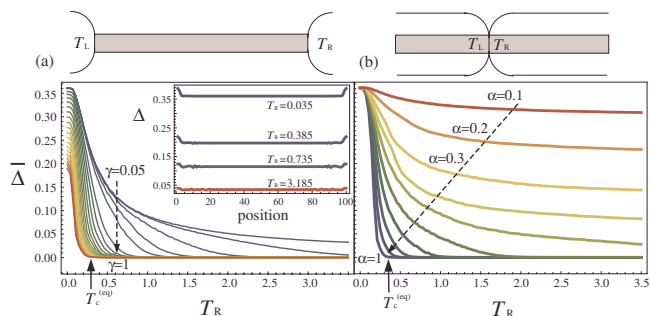


FIG. 1. (Color online) (a) Main panel: $\bar{\Delta}$ as a function of T_R (in units of the hopping parameter t) for different values of γ . At small values of γ , T_R can greatly exceed the equilibrium critical temperature (indicated by a solid arrow) without destroying superconductivity in the wire. Inset: position dependence of the order parameter for different values of T_R (at $\gamma=0.05$), demonstrating that it is uniform along the sample. (b) Same as in (a) but for a constant $\gamma=0.083$ and different values of α (describing the contact area of the right heat bath with the sample). Upper panel: the geometries considered in (a) and (b).

where $u_n(i)$ and $v_n(i)$ are quasiparticle and quasihole wave functions, respectively. This yields a set of eigenvalue equations for the quasiparticle (QP) wave functions,¹³

$$\begin{pmatrix} \hat{\xi} & \hat{\Delta} \\ \hat{\Delta}^* & -\hat{\xi}^* \end{pmatrix} \begin{pmatrix} u_n(i) \\ v_n(i) \end{pmatrix} = E_n \begin{pmatrix} u_n(i) \\ v_n(i) \end{pmatrix}, \quad (1)$$

where $\hat{\xi}u_n(i) = -t\sum_{\langle i,j \rangle} u_n(j) + (\epsilon_i - \mu + Un_i/2)u_n(i)$ (here n_i is the local electron density), and $\hat{\Delta}u_n(i) = \Delta_i u_n(i)$. From Eq. (1) one obtains the QP wave functions $u_n(i)$ and $v_n(i)$ and the energies E_n . In equilibrium, this procedure results in a closed self-consistent set of equations for the local SC order parameter and density, which were recently used to study, e.g., effects of disorder and magnetic fields in two-dimensional superconductors.^{14–16}

Since the QP excitations are noninteracting Fermions, they can be treated by the formalism of Refs. 9–11, which is aimed at studying such particles out of equilibrium. To generalize the method of Refs. 9–11 to the QP excitations, we define a single-particle density matrix $\hat{\rho}$, with matrix elements $\rho_{nn'} = \langle \gamma_{n\uparrow}^\dagger \gamma_{n'\uparrow} \rangle$ (note that the particle-hole symmetry allows one to treat only the up-spin excitations). The master equation for $\hat{\rho}$ is of the Lindblad form¹⁷ (setting $\hbar = k_B = 1$ hereafter) $\dot{\rho} = -i[\mathcal{H}, \rho] + \mathcal{L}_L[\rho] + \mathcal{L}_R[\rho]$, where \mathcal{H} is the diagonal matrix of energies E_n and

$$\mathcal{L}_{(L,R)}[\rho] = \sum_{nn'} \left(-\frac{1}{2} \{ V_{nn'}^{(L,R)\dagger} V_{nn'}^{(L,R)}, \rho \} + V_{nn'}^{(L,R)} \rho V_{nn'}^{(L,R)\dagger} \right) \quad (2)$$

describe environment-induced inelastic transitions between different single-particle states. The V operators in Eq. (2) take on a local form¹⁰

$$V_{nn'}^{(L,R)} = \sqrt{\Gamma_{nn'}^{(L,R)} f_D^{(L,R)}(E_n)} \gamma_{n\uparrow}^\dagger \gamma_{n'\uparrow},$$

$$\Gamma_{nn'}^{(L,R)} = \left| \Gamma_0 \sum_{r_i \in S_{L,R}} (u_n(r_i) u_{n'}^*(r_i) + v_n(r_i) v_{n'}^*(r_i)) \right|, \quad (3)$$

where $S_{L,R}$ are the contact area of the left (right) heat bath with the sample, $f_D^{(L,R)}(E_n) = 1/[1 + \exp(E_n/T_{L,R})]$ is the Fermi distribution and Γ_0 is some constant scattering rate. (We take $\Gamma_0 = 0.1$, changing Γ_0 does not alter the results presented above.) Note that the V operators act on the QPs and not on the electron operators, since the Fermi function is defined for the occupation of the QPs (to put it differently, those electrons which are in the superfluid phase do not experience scattering from the baths, only the QPs do).

Once the V operators are evaluated, the master equation is solved in the asymptotic time limit (i.e., $\dot{\rho} = 0$) and a solution for ρ is obtained. From this solution, the local density and order parameter are evaluated via the self-consistency condition, which reads

$$\Delta_i = U \sum_n u_n(i) v_n^*(i) (1 - 2\rho_{nn}),$$

$$n_i = 2 \sum_n [|u_n(i)|^2 \rho_{nn} + |v_n(i)|^2 (1 - \rho_{nn})]. \quad (4)$$

With Δ_i and n_i determined, the whole procedure (i.e., the solution of the BdG equations, the evaluation of the V operators and the solution of the master equation) is repeated until Δ_i and n_i no longer change (within the numerical tolerance of $< 10^{-5}$). We point that this treatment is of a mean-field type, and as such neglects statistical fluctuations in the Hamiltonian¹⁸ or phase slips.¹⁹ As we will show below, in the ideal case one can define an effective temperature of the wire, and hence in such a case one can follow the usual treatment of phase slips in SC wires, using the effective temperature as input.²⁰

We begin by presenting the averaged order parameter $\bar{\Delta} = \frac{1}{N} \sum_i \Delta_i$, where N is the total number of lattice sites. We consider the geometry shown in the side panel of Fig. 1(a), where the SC wire (gray area) is connected to the thermal baths only at its edges. The numerical parameters are as follows. The wire dimensions are 100×10 , $U=2$, $W=0$ (clean system) and the density is held at $n=0.875$ (i.e., n electrons per site on average, and the chemical potential is chosen self-consistently to maintain this filling). We define T_L by its ratio with the critical temperature at equilibrium T_c^{eq} , $T_L/T_c^{eq} = \gamma < 1$.

If Fig. 1(a), $\bar{\Delta}$ is plotted as a function of T_R for different values of γ , $\gamma=0.05$, (top curve), $0.1, \dots, 1$ (bottom curve). For low values of γ , $T_{R,c}$ can greatly exceed T_c^{eq} (marked by a solid arrow). In the inset of Fig. 1(a) we plot the local order parameter as a function of position along the wire (averaged over the transverse direction), at $\gamma=0.05$ for different values of $T_R=0.035, 0.385, 0.735$, and 3.185 (in units of t). We find that although there are two different temperatures at the edges, the order parameter is practically uniform along the wire, in agreement with the results of Refs. 10 and 11.

In Fig. 1(b) we study a somewhat different (and perhaps more realistic) situation, in which the heat baths are in contact with the wire not only at the edges but over some area [see side panel of Fig. 1(b)]. We define the parameter $\alpha(1-\alpha)$ to be the ratio between the contact area of the right (left) heat bath and the area of the whole wire, such that $\alpha=1$ stands for a system in full contact only with the right heat bath. In Fig. 1(b), $\bar{\Delta}$ is plotted as a function of T_R (at $\gamma=0.083$) for different values of α , $\alpha=0.1$, (top curve), $0.2, \dots, 1$ (bottom curve). The $\alpha=1$ curve is the equilibrium curve (with T_c^{eq} marked by an arrow). As seen, for different values of α , $T_{R,c}$ may again exceed T_c^{eq} . Also in this case we found that both $\bar{\Delta}$ and the local temperature (calculated for a similar one-dimensional geometry, with the method of Refs. 10 and 11) are uniform in space (not shown). This emphasizes the fact that the temperature is defined not by the local baths, but rather by the (inelastic) scattering between states, which in the clean case span the entire system. We also point out that one can *define* an effective local temperature as the one which relates to the local gap from the equilibrium

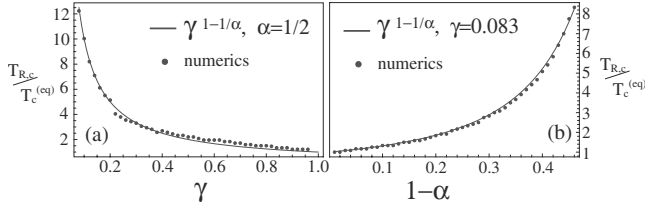


FIG. 2. (a) The ratio $T_{R,c}/T_c^{eq}$ as a function of γ , taken from the data of Fig. 1(a). The points correspond to the numerical data and the solid line is Eq. (6) with $\alpha=0.5$. (b) $T_{R,c}/T_c^{eq}$ as a function of α , taken from the data of Fig. 1(b). The points correspond to the numerical data and the solid line is Eq. (6) with $\gamma=0.083$.

theory of superconductivity¹³ (see below). Such a temperature (which may differ from the one measured by other means as in Ref. 11) is also uniform in clean wires, and is in principle experimentally accessible via scanning tunnel microscope measurements of the local tunneling density of states.

If one could simply define a local temperature which gradually shifts from T_L to T_R , then one would expect that T_R could not exceed T_c^{eq} and that the order parameter would not be uniform in space. In order to explain our findings, we recall that one of the main results of Refs. 10 and 11 is that in the ballistic limit the temperature is uniform, and a non-equilibrium DF develops, which is the average of the two DFs of the left and right baths. In the case represented in Fig. 1(b), we find that a weighted average between the DFs of the left and right baths develops, the weight being α . This result, along with the observation that Δ_i is uniform in space, allows us to find an analytical expression for the effective $T_{R,c}$ as follows. In the equilibrium theory of superconductivity,¹³ the critical temperature T_c^{eq} is determined by the gap equation $\frac{1}{N_0 U} = \frac{1}{2} \int_0^{\omega_D/T_c^{eq}} \frac{1}{x} [1 - 2f(x)] dx$, where N_0 is the density of states at the Fermi energy, ω_D is the Debye frequency and $f(x)$ is the DF. From the above discussion, in the nonequilibrium case we have $f(x) = \alpha f_D^{(R)}(x) + (1-\alpha) f_D^{(L)}(x)$. The resulting equation for $T_{R,c}$ then reads

$$\frac{1}{N_0 U} = \frac{\alpha}{2} \int_0^{\omega_D/T_{R,c}} \frac{1}{x} [1 - 2f(x)] dx + \frac{1-\alpha}{2} \int_0^{\omega_D/T_L} \frac{1}{x} [1 - 2f(x)] dx. \quad (5)$$

These integrals may be evaluated exactly, and with $T_L/T_c^{eq} = \gamma$ we find

$$\frac{T_{R,c}}{T_c^{eq}} = \gamma^{1-1/\alpha}. \quad (6)$$

In Fig. 2 we plot $T_{R,c}/T_c^{eq}$ as a function of γ [Fig. 2(a)] and of α [Fig. 2(b)], taken from the data of Figs. 1(a) and 1(b), respectively. The solid line corresponds to Eq. (6) for the two cases, with the corresponding parameters taken from the numerical calculation. The agreement between Eq. (6) and the numerical results confirms that indeed the DF is a weighted average of the DFs of the two baths. One can now use Eq. (6) to estimate the effective $T_{R,c}$ of real materials. As

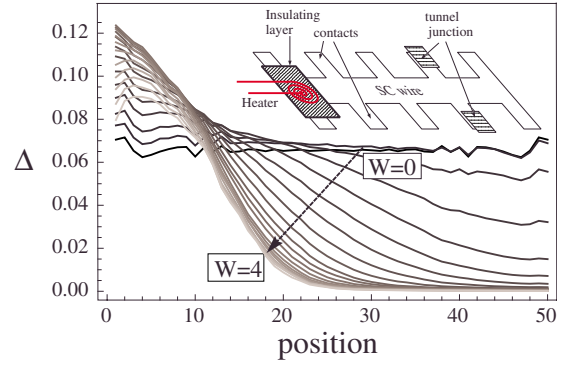


FIG. 3. (Color online) Position dependence of the order parameter Δ , calculated with increasing values of disorder, from the clean case $W=0$ (dark line) to strongly disordered case $W=4$ (light gray line), averaged over 500 realizations of disorder. For this example $T_L=0.02$, $T_c^{eq}=0.2$, and $T_R=1.5$. Inset: suggested experimental setup. An insulating layer is deposited on top of a SC wire with etched contacts, and a heater coil is placed on top of it, to generate local heating. The resistance is then monitored along the other contacts (which may be either four-terminal contacts or tunnel junctions).

a practical use, it is advantageous to raise $T_{R,c}$ above the freezing point of liquid Nitrogen, ~ 77 K. For example, consider a desired working temperature of ~ 80 K, and local refrigerators which cool down to 40 K, deposited on a SC wire with $T_c^{eq} \sim 60$ K. A coverage of 40% refrigerators would increase $T_{R,c}$ to 78 K.

Perhaps a more intriguing possibility is the enhancement of $T_{R,c}$ to room temperature. For a wire made of the newly found iron compound²¹⁻²³ ($T_c^{eq} \sim 50$ K) heated (or cooled) at the edges ($\alpha=0.5$), a temperature $T_L=7.5$ K would drive $T_{R,c}$ above room temperature. For nanoscale wires made of a high- T_c material, the fabrication of which was recently demonstrated,²⁴ taking $T_c^{eq} \sim 80$ K, local cooling of $T_L=20$ K and coverage of $\alpha=0.5$ would drive $T_{R,c}$ above room temperature. Of course, other effects (e.g., phonon scattering, phase fluctuations etc.) may increase in importance for such high values of T_R , and thus inhibit the zero-resistance state.

Next we turn to study the effect of disorder. In Ref. 11 it was shown that the form of the nonequilibrium DF is robust against disorder, but that the local temperature profile changes from a constant temperature to a position-dependent profile. In Fig. 3 we plot the order parameter Δ_i as a function of position along the wire for $T_R=1.5$ (which is above $T_c^{eq} \sim 0.2$ in this example), $T_L=0.02$ and dimensions 50×10 , for different values of disorder, $W=0$ (dark curve), $0.2, \dots, 4$ (bright curve). The order parameter is averaged over 500 realizations of disorder. For a clean system, Δ is uniform and assumes a position dependence with increasing disorder. For small values of disorder it is finite everywhere in the sample, in agreement with Anderson's theorem.⁸ For large values of disorder it vanishes near the right edge, and increases at the left edge. This is due to the fact that a local temperature ensues which varies from the left to the right temperature. In fact, if one compares Fig. 3 to the local temperature calculated for similar (normal) systems (Fig. 1 of Ref. 11) one finds that Δ obeys a simple BCS-like law, with the temperature replaced by the local temperature.

We point out that in this case the local temperature near the left edge (which corresponds to T_L) is lower than the effective temperature of the clean sample, and hence the rise in the value of Δ with increasing disorder near the left edge. The distance from the right edge at which Δ vanishes indicates the “thermal length” where the local temperature is close to T_R .¹¹ We also note that already for relatively small values of disorder ($W \approx 0.5$ in our case) the localization length is smaller than the system size, which means that the onset of a vanishing gap in the system (occurring at $W \approx 1.4$) does not directly correspond to the onset of localization.^{11,25}

In order to test our predictions, we suggest the experimental setup shown in the inset of Fig. 3. It consists of a SC wire with etched contacts (either regular four-terminal contacts, or made as tunnel junction, aimed at measuring the local density of states at that location). On top of one of the contacts an insulating layer is deposited, and on top of it a heater coil is set. The temperature below the heater can be calibrated by measuring the resistance between the contacts beneath the heater when the wire is in the normal state. Then the whole device is cooled down, and the resistance through the other contacts is measured as a function of the current that passes through the heater (i.e., the local temperature beneath it). In a uniform system, the resistances of all the contacts should vanish if the system is SC, and have finite values once the local temperature beneath the heater rises above $T_{R,c}$. If the system is disordered, the different contacts should exhibit a gradual transition to a normal state.

The length-scale that determines the onset of a temperature gradient is in this case the QP mean-free path (or diffusion length).²⁵ While it can be estimated for low T_c metals, for high- T_c materials it is unknown (although it is suspected

to be small, based on their poor conduction in the normal state). By controlling the distances between the contacts our proposed experiment can thus serve to determine this length, by relating it to the length scale at which a temperature gradient develops.

In this work we have neglected the effect of phonon scattering, and assumed the effective electron-electron interaction is not appreciably changed by temperature, supported by the fact that the Debye temperature is much larger than T_c . However, considering our geometry, electron-phonon (e-ph) interaction effects may play a significant role as T_R reaches the Debye temperature. This is probably more important in disordered wires (for clean wires we expect that, on equal grounds, phonons will also acquire a uniform temperature). The e-ph interaction may induce inelastic electron transitions, which will reduce the inelastic mean-free path. Since the temperature profile is sensitive to the inelastic mean-free path,²⁵ this effect may be seen in our suggested experiment.

To conclude, we point out that in recent years there have been tremendous advances in fabricating microrefrigerators, based on the thermoelectric Peltier effect, and which can locally cool down their environment substantially.^{26,27} Since the efficiency of thermoelectric materials is likely to increase in the future,²⁸ one can conceive a device composed of a (relatively) high- T_c material, on top of which are embedded a series of microrefrigerators, covering an area of the material and cooling it enough for it to operate at a temperature which exceeds its T_c ; a possibility that may allow for integrating superconducting wires as circuit elements in various devices.

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²⁰Phase fluctuations are unlikely to change the local effective temperature since these are excitations that do not carry heat (as opposed to QP excitations). In the disordered case, such a treatment cannot work as the temperature becomes space dependent and one needs a different formalism to treat phase fluctuations in the presence of a temperature gradient. Such a theory is beyond the scope of the present work and is left for future studies.

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